

一类非线性二自由度弹簧振子的动力学仿真研究

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摘要: **目的** 研究二自由度正切型系统的动态特性, 为缓冲包装设计提供理论依据。 **方法** 建立一类正切非线性缓冲系统二自由度振子模型, 采用数值仿真手段讨论质量比、频率比、易损件阻尼、缓冲材料阻尼等对振子变形响应的影响规律。 **结果** 质量比和频率比对最大变形影响非常小, 缓冲材料阻尼对最大变形的影响最显著, 是控制材料变形的主要因素。 **结论** 通过改变缓冲材料的阻尼来控制缓冲垫的变形, 完成相应的缓冲包装设计。

关键词: 正切; 二自由度振子; 冲击; 位移响应

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Dynamic Simulation of One Class of Two-degree-of-freedom Nonlinear Spring Oscillator

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ABSTRACT: Objective To provide a theoretical basis for the design of cushioning packaging, the dynamic characteristics of two-degree-of-freedom tangent system was studied. **Methods** A two-degree-of-freedom vibration system was proposed to model the shock response of a class of tangent cushioning system. The effect rules of the mass ratio, the frequency ratio, the damping ratio of joint and the cushion on the maximum response displacement of cushion were studied. **Results** The results showed that the damping ratio of cushion was the key factor in controlling material deformation due to its noticeable influence on the response, and the effects of mass and frequency ratio were very little. **Conclusion** Deformation of the buffer pad could be controlled through changing the damping of the buffer material to complete the corresponding cushioning packaging design.

KEY WORDS: tangent; two-degree-of-freedom vibration system; shock pulse; response displacement

缓冲包装是确保产品物流安全的重要措施, 当前研究主要基于经典的产品脆值及破损边界理论开展^[1-2]。考虑到绝大多数缓冲包装材料是强非线性, 王志伟^[3]研究了各种非线性缓冲系统(三次非线性、双曲正切型等)在不同典型激励(矩形、半正弦脉冲等)下的响应特征, 同时还以包装系统的特征参数和无量纲化的初始跌落速度作为评价量。王军、王志伟^[4]研究了多层堆码包装系统在半正弦脉冲激励下的冲击特性, 建立了多层堆码包装系统动力学模型, 得到系统

统一动力学方程并进行求解。卢立新^[5-6]以果蔬类产品作为研究对象, 提出了跌落损伤脆值的新概念, 构建了果品类产品的损伤边界曲线。陈安军^[7-8]以斜支承弹簧系统为研究对象, 建立了矩形脉冲激励下的系统非线性动力学方程, 以系统加速度响应峰值与脉冲激励幅值之比反映系统在冲击作用下的响应指标, 以脉冲激励时间为变量建立系统的冲击谱, 讨论了脉冲激励幅值以及系统阻尼等对冲击谱的影响规律。王军^[9-10]基于产品关键部件建立非线性包装系统的破损

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边界曲面的概念。高德、卢富德^[11-12]考虑包装件跌落过程中的转动因素,建立了正切及其组合的非线性多自由度系统的冲击模型,并获得了响应的数值解。高德^[13]依据粘弹性力学理论,考虑实际使用的发泡聚乙烯缓冲材料,建立了非线性多自由度缓冲包装系统模型,并运用混合罚函数方法,对多自由度缓冲包装系统进行优化设计,计算结果具有较高精度。段宁宁^[14]等以易损件的斜支承包装系统为研究对象,建立了矩形脉冲激励下系统无量纲非线性冲击动力学方程并求解,分析讨论了系统支承角、脉冲激励幅值以及质量比等对冲击响应谱的影响规律。宋爽^[15]以悬挂式弹簧系统为研究对象,建立跌落工况无阻尼条件下系统的无量纲几何非线性动力学方程。

文中初步以包装材料中常见的正切系统为例,分析在运输半正弦冲击载荷下产品与易损件质量比、频率比,易损件与缓冲材料的阻尼对缓冲材料最大变形量的影响规律。

1 模型与方程

产品包装系统模型见图1,假设被包装产品包括产品主体质量 m_2 和易损件质量 m_1 ,它们之间用非线性弹簧连接,弹簧的刚度和阻尼分别表示为 k_1, c_1 ,缓冲材料的刚度和阻尼分别表示为 k_2, c_2 ,并且满足:

$$f(x) = \frac{2k_2 d_b}{\pi} \tan \frac{\pi}{2d_b} x + c_2 \dot{x} \quad (1)$$

式中: $f(x)$ 为非线性弹簧的反作用力; x 与 \dot{x} 为非线性弹簧的实际变形量与变形速率; d_b 为非线性弹簧的实际最大允许变形量。

假设该系统在半正弦冲击载荷下运动,半正弦脉冲信号数学函数为:

$$u'' = u_{0m}'' \sin \frac{\pi t}{t_0} \Delta H(t, t_0) \quad (2)$$

式中: u'' 为半正弦冲击加速度; u_{0m}'' 为冲击加速度的最大值; t 为半正弦冲击的任意时刻; t_0 为半正弦冲击所经历的时间; $\Delta H(t, t_0)$ 为脉冲函数,

$$\Delta H(t, t_0) = \begin{cases} 0 & 0 \leq t \leq t_0 \\ 1 & t > t_0 \end{cases} .$$

依据以上分析,建立包装系统二自由振子模型的运动学方程:

$$m_1 \ddot{x}_1 = k_1(x_2 - x_1) + c_1(\dot{x}_2 - \dot{x}_1)$$

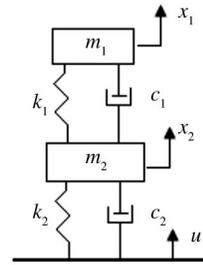


图1 易损件-产品-包装二自由度振子模型

Fig.1 Two-degree-oscillator model for critical component-product-packaging system

$$m_2 \ddot{x}_2 = \frac{2k_2 d_b}{\pi} \tan \left[\frac{\pi}{2d_b} (u - x_2) \right] + c_2(\dot{u} - \dot{x}_2) - k_1(x_2 - x_1) + c_1(\dot{x}_2 - \dot{x}_1) \quad (3)$$

初始条件为: $x_{1,2}(0) = 0, \dot{x}_{1,2}(0) = 0$ 。引入包装主体与易损件之间,包装主体与地面之间的无量纲化的相对位移; $\delta_1 = \frac{x_2 - x_1}{L}, \delta_2 = \frac{u - x_2}{L}, \tau = \frac{t}{T}$, 并令包装主体时间系数 $T = \sqrt{m_2/k_2}$, 弹簧长度系数

$L = \frac{2d_b}{\pi}$, 易损件固有频率 $\omega_1 = \sqrt{\frac{k_1}{m_1}}$, 包装主体固有

频率 $\omega_2 = \sqrt{\frac{k_2}{m_2}}$, 易损件阻尼比 $\zeta_1 = \frac{c_1}{2\sqrt{m_1 k_1}}$, 缓冲

垫阻尼比 $\zeta_2 = \frac{c_2}{2\sqrt{m_2 k_2}}$, 易损件和主体件频率比

$\lambda_1 = \frac{\omega_1}{\omega_2}$, 易损件和主体件质量比 $\lambda_2 = \frac{m_1}{m_2}$;

$\ddot{\delta}_1 = \tan \delta_2 + 2\zeta_2 \dot{\delta}_2 - (\lambda + 1) \frac{\omega_1^2}{\omega_2^2} \delta_1 - 2(\lambda + 1) \frac{\omega_1}{\omega_2} \zeta_1 \dot{\delta}_1$

$\ddot{\delta}_2 = \ddot{u} + \lambda \frac{\omega_1^2}{\omega_2^2} \delta_1 + 2\lambda \frac{\omega_1}{\omega_2} \zeta_1 \dot{\delta}_1 - \tan \delta_2 - 2\zeta_2 \dot{\delta}_2 \quad (4)$

初始条件为: $\delta_{1,2}(0) = 0, \dot{\delta}_{1,2}(0) = 0$ 。 $\ddot{U} = \beta u_{0m}'' \cdot \sin \frac{\pi \tau}{\tau_0} \Delta H(\tau, \tau_0)$, 其中, β 为系统特性参量 $\beta = \frac{T^2}{L}$ 。

2 系统冲击响应影响因素

文中应用四阶R-K方法获得方程(4)的数值解,并研究各参数对缓冲垫最大变形量 δ_{2m} 的影响

规律。

2.1 质量比

当 $\zeta_1=0, \zeta_2=0, \beta u_{0m}'' = 0.5, \tau_0=0.5$ 时,易损件和主体质量比对缓冲材料最大变形的影响见图2。

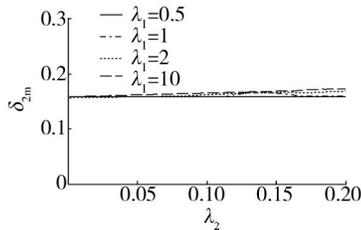


图2 质量比对缓冲材料最大变形的影响

Fig.2 Effect of mass ratio on the maximum response displacement of the cushion

2.2 频率比

当 $\zeta_1=0, \zeta_2=0, \beta u_{0m}'' = 0.5, \tau_0=0.5$ 时,易损件和主体频率比对缓冲材料最大变形的影响见图3。

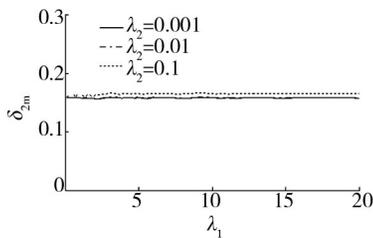


图3 频率比对缓冲材料最大变形的影响

Fig.3 Effect of frequency ratio on the maximum response displacement of the cushion

2.3 易损件阻尼

当 $\zeta_2=0, \lambda_2=0.1, \beta u_{0m}'' = 0.5, \tau_0=0.5$ 时,易损件阻尼对缓冲材料最大变形的影响见图4。

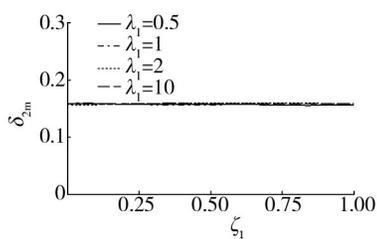


图4 易损件阻尼对缓冲材料最大变形的影响

Fig.4 Effect of damping ratio of the joint on the maximum response displacement of the cushion

2.4 缓冲材料阻尼

当 $\zeta_1=0, \lambda_2=0.1, \beta u_{0m}'' = 0.5, \tau_0=0.5$ 时,材料阻尼等对缓冲材料最大变形的影响见图5。

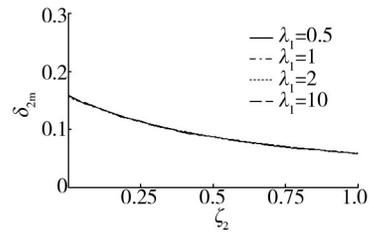


图5 缓冲材料阻尼对缓冲材料最大变形的影响

Fig.5 Effect of damping ratio of cushion on the maximum response displacement of the cushion

3 结语

经分析可得,质量比对缓冲垫最大变形的影响不显著;频率比对缓冲垫最大变形的影响不显著;随着材料阻尼的增大,缓冲垫的最大变形减小;连接部阻尼对主体峰值响应加速度和缓冲垫的最大变形影响不显著。

文中研究了质量比、频率比、易损件阻尼、缓冲垫阻尼等对二自由度正切包装系统振子在运输冲击载荷下缓冲材料最大位移响应的影响。结果表明,材料阻尼是控制材料最大变形的主要因素,研究结论可作为缓冲包装设计的理论依据。

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(上接第12页)

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